INITIAL MODEL OF TRANSPORT MEANS AVAILABILITY OF REAL CITY TRANSPORT SYSTEM

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Abstract

The article presents a determination method of transport means serviceability in a real city transport system. The considerations have been demonstrated on the basis of a real city bus transport system in a chosen agglomeration. The executive subsystem consisting of elementary subsystems of the type human – technical object (operator – transport means) whose serviceability and reliability have a direct influence on capability to perform the transport task is directly responsible for accomplishment of the city transport system tasks.

In order to develop a model of means of transport serviceability there were determined significant states of the operational use process and there was made a division and reduction of the number of states in terms of the availability criterion. On this basis, an event model of the process of transport means operational use, and then a mathematical model of this process, were built with the assumption that its model is a homogenous Markov's process X(t). Next, boundary values of availability coefficient for the distinguished availability levels of transport means used in the city transport system were determined for operational data obtained from a real city bus transport system.

The discussed model for determination of transport means availability is an initial one, developed on the basis of a nine-state model of operational use. In further part of the paper, there will be developed a resultant model of transport means availability determination (semi-Markov's sixteen-state model), being a component of a wider, decision model for creation and assessment of transport means availability.

Keywords: *city transport system, Markov's model, technical object availability*

1. Introduction

The concept of availability as the system characteristic is used for analysis of emergency systems, that is, air forces, ambulance service, fire brigade, transport systems. In these systems, in case of emergency, a human or a group of humans together with technical objects assigned to them, are supposed to start carrying out these tasks immediately. Generally, the technical object availability should be understood as its feature (system or element) which characterizes it in terms of its capability to promptly reach and maintain availability (enabling accomplishment of the task) [5, 6, 8, 9].

Transport systems are systems whose main goal is to carry out transport tasks. The size of transport tasks is defined by the frequency of rides and the quantity of transported loads in a given time, on a given route. Direct realization of the transport system task is the responsibility of an executive system consisting of elementary subsystems of the type human – technical object (operator – means of transport) whose availability and reliability has a significant influence on the capability of the transport task accomplishment. Availability and reliability of means of transport is maintained at a proper level thanks to realization of serviceability assurance processes, carried out within the executive system by a breakdown gang and within the logistic system on repair stands of a depot. From the above, it follows that the transport system availability depends on the possibility of proper control of the operational use process, both in the executive and logistic subsystems.

The model of the technical object availability determination (means of transport), presented in this paper, is the first stage of building a decision model for the transport system availability control.

2. Event model of the operational use process

The model of operational use was built on the basis of an analysis of spaces of operational use states and events concerning technical objects (city buses), used in the analyzed real transport system. In result of identification of the analyzed transport system and a multi-state process of technical objects operational use realized within it, there were determined the process significant operational use states and possible transitions between the distinguished states. On this basis, a graph of the operational use process state changes was built which is presented in Fig. 1.

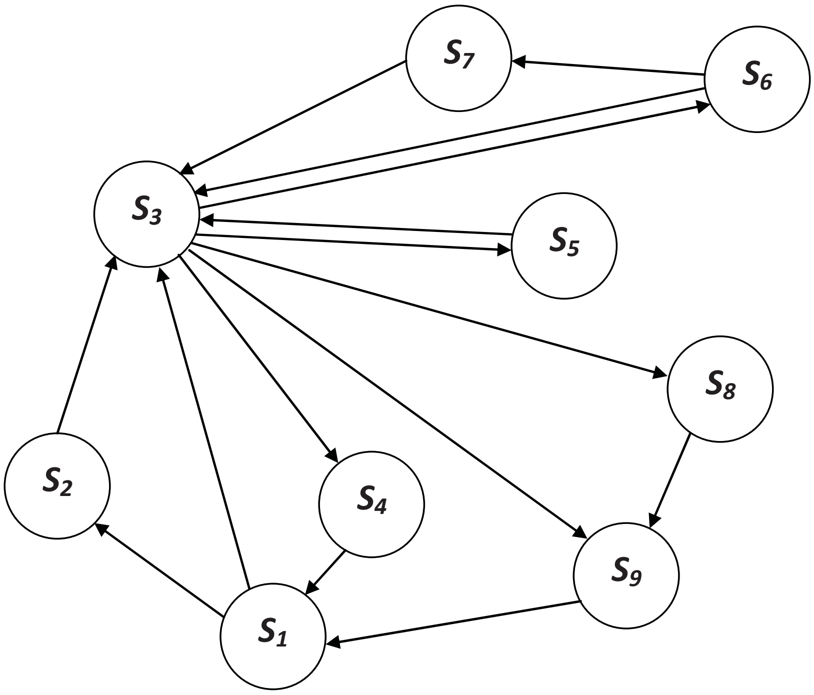

Fig. 1 Directed graph representing means of transport operation and maintenance process: S_1 – stay on the depot hardstand, S_2 – serviceability assurance on the depot hardstand, S_3 – transport task accomplishment, S_4 – refueling between rush hours, S_5 – serviceability assurance by a breakdown truck without loss of ride, S_6 – serviceability assurance by a breakdown truck with loss of ride, S_7 – waiting for the task to be carried out after serviceability assurance by a breakdown truck, S_8 – emergency exit, S_9 – serviceability assurance on stands of the serviceability assurance subsystem

3. Mathematical model of the operational use process

In result of the carried out analysis of assumptions and limitations, it was accepted that the technical object operational use model is Markov's process $X(t)$. Using Markov's process for mathematical modelling of the operational use process, the following assumptions were accepted:
- Markov's process $X(t)$ reflects the modelled real operational use process well enough from the point of view of the research goal,
- the modeled process of operational use has a definite number of states S_i, $i = 1, 2,\dots, 9$,
- random process $X(t)$ which is a mathematical model of the operational use process is homogenous,
- in time $t = 0$, the process is in state S_i (initial state is S_i state).

On the basis of an oriented graph, presented in Fig. 1, there was built a matrix of P states change probabilities and matrix of Λ states change intensity of $X(t)$ process:

$$P = \begin{bmatrix} 0 & p_{1,2} & p_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{3,4} & p_{3,5} & p_{3,6} & 0 & p_{3,8} & p_{3,9} \\ p_{4,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{5,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{6,3} & 0 & 0 & 0 & p_{6,7} & 0 & 0 \\ 0 & 0 & p_{7,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{8,9} \\ p_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (1)$$

$$\Lambda = \begin{bmatrix} -\lambda_{1,1} & \lambda_{1,2} & \lambda_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_{2,2} & \lambda_{2,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_{3,3} & \lambda_{3,4} & \lambda_{3,5} & \lambda_{3,6} & 0 & \lambda_{3,8} & \lambda_{3,9} \\ \lambda_{4,1} & 0 & 0 & -\lambda_{4,4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{5,3} & 0 & -\lambda_{5,5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{6,3} & 0 & 0 & -\lambda_{6,6} & \lambda_{6,7} & 0 & 0 \\ 0 & 0 & \lambda_{7,3} & 0 & 0 & 0 & -\lambda_{7,7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{8,8} & \lambda_{8,9} \\ \lambda_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{9,9} \end{bmatrix}, \qquad (2)$$

where

p_{ij} - probability of transition from state S_i to state S_j,

λ_i - intensity of stay in state S_i of $X(t)$ process,

λ_{ij} - intensity of transition from state S_i to state S_j of $X(t)$ process.

On the basis of Λ (2) matrixes, in order to determine boundary probabilities p_i^* of stay in Markov's process states, a system of linear equations was developed

$$\sum_{i=1}^{9} \lambda_{ij} \cdot p_i^* = 0, \quad j = 1,2,...,9, \qquad (3)$$

hence, according to (1):

$$\begin{cases} -\lambda_{11} \cdot p_1^* + \lambda_{41} \cdot p_4^* + \lambda_{91} \cdot p_9^* = 0 \\ \lambda_{12} \cdot p_1^* - \lambda_{22} \cdot p_2^* = 0 \\ \lambda_{13} \cdot p_1^* + \lambda_{23} \cdot p_2^* - \lambda_{33} \cdot p_3^* + \lambda_{53} \cdot p_5^* + \lambda_{63} \cdot p_6^* + \lambda_{73} \cdot p_7^* = 0 \\ \lambda_{34} \cdot p_3^* - \lambda_{44} \cdot p_4^* = 0 \\ \lambda_{35} \cdot p_3^* - \lambda_{55} \cdot p_5^* = 0 \\ \lambda_{36} \cdot p_3^* - \lambda_{66} \cdot p_6^* = 0 \\ \lambda_{67} \cdot p_6^* - \lambda_{77} \cdot p_7^* = 0 \\ \lambda_{38} \cdot p_3^* - \lambda_{88} \cdot p_8^* = 0 \\ \lambda_{39} \cdot p_3^* + \lambda_{89} \cdot p_8^* - \lambda_{99} \cdot p_9^* = 0 \end{cases} . \qquad (4)$$

Equation system (4) is a dependent system. In order to solve this system, one of the equations (third equation) was replaced with a normalizing condition in the form

$$\sum_{i=1}^{9} p_i^* = 1 . \tag{5}$$

Then, equation system (4) regarding condition (5) was recorded in the matrix form:

$$\begin{bmatrix} -\lambda_{1,1} & 0 & 0 & \lambda_{4,1} & 0 & 0 & 0 & 0 & \lambda_{9,1} \\ \lambda_{1,2} & -\lambda_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & \lambda_{3,4} & -\lambda_{4,4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{3,5} & 0 & -\lambda_{5,5} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{3,6} & 0 & 0 & -\lambda_{6,6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_{6,7} & -\lambda_{7,7} & 0 & 0 \\ 0 & 0 & \lambda_{3,8} & 0 & 0 & 0 & 0 & -\lambda_{8,8} & 0 \\ 0 & 0 & \lambda_{3,9} & 0 & 0 & 0 & 0 & \lambda_{8,9} & -\lambda_{9,9} \end{bmatrix} \cdot \begin{bmatrix} p_1^* \\ p_2^* \\ p_3^* \\ p_4^* \\ p_5^* \\ p_6^* \\ p_7^* \\ p_8^* \\ p_9^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{6}$$

In result of solution of equation system (6) boundary probabilities p_i^* of stay in Markov's process states were obtained, described by the following dependencies:

$$p_1^* = \frac{1}{\lambda_{11}} \left\{ \left(\frac{\lambda_{41} \cdot \lambda_{34}}{\lambda_{44}} \right) + \left[\frac{\lambda_{91}}{\lambda_{99}} \cdot \left(\frac{\lambda_{38} \cdot \lambda_{89}}{\lambda_{88}} + \lambda_{39} \right) \right] \right\} \cdot p_3^* , \tag{7}$$

$$p_2^* = \frac{\lambda_{12}}{\lambda_{11} \cdot \lambda_{22}} \left\{ \left(\frac{\lambda_{41} \cdot \lambda_{34}}{\lambda_{44}} \right) + \left[\frac{\lambda_{91}}{\lambda_{99}} \cdot \left(\frac{\lambda_{38} \cdot \lambda_{89}}{\lambda_{88}} + \lambda_{39} \right) \right] \right\} \cdot p_3^* , \tag{8}$$

$$p_3^* = \frac{1}{1 + \left\langle \frac{\lambda_{12}}{\lambda_{11} \cdot \lambda_{22}} \left\{ \left(\frac{\lambda_{41} \cdot \lambda_{34}}{\lambda_{44}} \right) + \left[\frac{\lambda_{91}}{\lambda_{99}} \cdot \left(\frac{\lambda_{38} \cdot \lambda_{89}}{\lambda_{88}} + \lambda_{39} \right) \right] \right\} \right\rangle \cdot \left(1 + \frac{\lambda_{12}}{\lambda_{22}} \right) + \frac{\lambda_{34}}{\lambda_{44}} + \frac{\lambda_{35}}{\lambda_{55}} + \frac{\lambda_{36}}{\lambda_{66}} \cdot \left(1 + \frac{\lambda_{67}}{\lambda_{77}} \right) + \frac{\lambda_{38}}{\lambda_{88}} \cdot \left(1 + \frac{\lambda_{89}}{\lambda_{99}} \right) + \frac{\lambda_{39}}{\lambda_{99}}} , \tag{9}$$

$$p_4^* = \frac{\lambda_{34}}{\lambda_{44}} \cdot p_3^* , \tag{10}$$

$$p_5^* = \frac{\lambda_{35}}{\lambda_{55}} \cdot p_3^* , \tag{11}$$

$$p_6^* = \frac{\lambda_{36}}{\lambda_{66}} \cdot p_3^* , \tag{12}$$

$$p_7^* = \frac{\lambda_{36} \cdot \lambda_{67}}{\lambda_{66} \cdot \lambda_{77}} \cdot p_3^* , \tag{13}$$

$$p_8^* = \frac{\lambda_{38}}{\lambda_{88}} \cdot p_3^* , \tag{14}$$

$$p_9^* = \frac{1}{\lambda_{99}} \left(\frac{\lambda_{38} \cdot \lambda_{89}}{\lambda_{88}} + \lambda_{39} \right) \cdot p_3^* . \tag{15}$$

4. Availability of technical objects (transport means) in a transport system

Generally, the technical object availability to carry out the assigned task is defined as its feature which characterizes it in terms of its capability to reach availability in time t of time period τ_r time reserve intended for the object equipment and/or serviceability assurance.

In the presented model there were distinguished three levels of the technical object availability (means of transport):

- $G^{24}_{OT(1)}$ - refers to technical objects which in a given moment t are fit for use and equipped, that is objects which carry out assigned tasks or wait to start accomplishment of the task on the hardstand of a depot or after repair by a breakdown truck,
- $G^{24}_{OT(2)}$ - refers to technical objects of $G^{24}_{OT(1)}$ level and technical objects which were equipped on the depot stands, in time $T_z \le \tau_z$ of the time reserve intended for equipping the technical object without losing a ride,
- $G^{24}_{OT(3)}$ - refers to technical objects of $G^{24}_{OT(2)}$ level and objects which were made serviceable on the hardstand of a depot or by a unit of a breakdown truck in time $T_u \le \tau_u$ of the time reserve intended for the technical object serviceability assurance without losing a ride.

The considered model assumes that both the equipment of the technical object during the reserve time τ_z and its serviceability assurance τ_u, do not involve disruption of the task accomplishment, thereby the necessity of replacement of the equipped or serviceability assured object with another one (standby object).

For the purpose of the transport system availability determination (transport means), operational states of the technical object should be divided into availability states S_G and unavailability states S_{NG} of the object to carry out the assigned tasks, on the basis of Markov's operational process model. The technical object availability states are defined as those states in which the object together with its operator stay in the transport system, being serviceable and equipped. Unavailability states are those states in which the object or the operator stay beyond the transport system (serviceable or unserviceable), and also when an unserviceable or unequipped object stays within the system.

For a general case, availability of the transport system technical objects is determined on the basis of Markov's model of the operational use process, and is defined as the sum of boundary probabilities p_i^* of stay in states belonging to the set of availability states

$$G = \sum_i p_i^*, \text{ for } \quad S_i \in S_G, \;\; i = 1,2,...,9 . \tag{16}$$

In the presented model, there have been distinguished the following states of the technical object availability for particular availability levels:

- for the first level:
  state S_1 - stay on the hardstand of a bus depot,
  state S_3 - carrying the transport task,
  state S_7 - waiting for beginning accomplishment of the transport task after serviceability assurance by a breakdown truck,
- for the second level:
  state S_1 - stay on the hardstand of a bus depot,
  state S_3 - carrying the transport task,
  state S_7 - waiting for beginning accomplishment of the transport task after serviceability assurance by a breakdown truck,
  state S_4 - refuelling between rush hours,

- for the third level:
  state S_1 - stay on the hardstand of a bus depot,
  state S_3 - carrying the transport task,
  state S_7 - waiting for beginning accomplishment of the transport task after serviceability assurance by a breakdown truck,
  state S_4 - refueling between rush hours,
  state S_2 - serviceability assurance on the depot hardstand,
  state S_5 - serviceability assurance by a breakdown truck unit without losing a course.

Then, the transport system technical object availability is described by the following dependencies:

$$G_{OT(1)}^{24} = p_1^* + p_3^* + p_7^*, \tag{17}$$

$$G_{OT(1)}^{24} = \frac{1+\frac{1}{\lambda_{11}}\left\{\left(\frac{\lambda_{41}\cdot\lambda_{34}}{\lambda_{44}}\right)+\left[\frac{\lambda_{91}}{\lambda_{99}}\cdot\left(\frac{\lambda_{38}\cdot\lambda_{89}}{\lambda_{88}}+\lambda_{39}\right)\right]\right\}+\frac{\lambda_{36}\cdot\lambda_{67}}{\lambda_{66}\cdot\lambda_{77}}}{1+\left\langle\frac{\lambda_{12}}{\lambda_{11}\cdot\lambda_{22}}\left\{\left(\frac{\lambda_{41}\cdot\lambda_{34}}{\lambda_{44}}\right)+\left[\frac{\lambda_{91}}{\lambda_{99}}\cdot\left(\frac{\lambda_{38}\cdot\lambda_{89}}{\lambda_{88}}+\lambda_{39}\right)\right]\right\}\right\rangle\cdot\left(1+\frac{\lambda_{12}}{\lambda_{22}}\right)+\frac{\lambda_{34}}{\lambda_{44}}+\frac{\lambda_{35}}{\lambda_{55}}+\frac{\lambda_{36}}{\lambda_{66}}\cdot\left(1+\frac{\lambda_{67}}{\lambda_{77}}\right)+\frac{\lambda_{38}}{\lambda_{88}}\cdot\left(1+\frac{\lambda_{89}}{\lambda_{99}}\right)+\frac{\lambda_{39}}{\lambda_{99}}}, \tag{18}$$

$$G_{OT(2)}^{24} = p_1^* + p_3^* + p_7^* + p_4^*, \tag{19}$$

$$G_{OT(2)}^{24} = \frac{1+\frac{1}{\lambda_{11}}\left\{\left(\frac{\lambda_{41}\cdot\lambda_{34}}{\lambda_{44}}\right)+\left[\frac{\lambda_{91}}{\lambda_{99}}\cdot\left(\frac{\lambda_{38}\cdot\lambda_{89}}{\lambda_{88}}+\lambda_{39}\right)\right]\right\}+\frac{\lambda_{36}\cdot\lambda_{67}}{\lambda_{66}\cdot\lambda_{77}}+\frac{\lambda_{34}}{\lambda_{44}}}{1+\left\langle\frac{\lambda_{12}}{\lambda_{11}\cdot\lambda_{22}}\left\{\left(\frac{\lambda_{41}\cdot\lambda_{34}}{\lambda_{44}}\right)+\left[\frac{\lambda_{91}}{\lambda_{99}}\cdot\left(\frac{\lambda_{38}\cdot\lambda_{89}}{\lambda_{88}}+\lambda_{39}\right)\right]\right\}\right\rangle\cdot\left(1+\frac{\lambda_{12}}{\lambda_{22}}\right)+\frac{\lambda_{34}}{\lambda_{44}}+\frac{\lambda_{35}}{\lambda_{55}}+\frac{\lambda_{36}}{\lambda_{66}}\cdot\left(1+\frac{\lambda_{67}}{\lambda_{77}}\right)+\frac{\lambda_{38}}{\lambda_{88}}\cdot\left(1+\frac{\lambda_{89}}{\lambda_{99}}\right)+\frac{\lambda_{39}}{\lambda_{99}}}, \tag{20}$$

$$G_{OT(3)}^{24} = p_1^* + p_3^* + p_7^* + p_4^* + p_2^* + p_5^*, \tag{21}$$

$$G_{OT(3)}^{24} = \frac{1+\frac{1}{\lambda_{11}}\cdot\left(1+\frac{\lambda_{12}}{\lambda_{22}}\right)\cdot\left\{\left(\frac{\lambda_{41}\cdot\lambda_{34}}{\lambda_{44}}\right)+\left[\frac{\lambda_{91}}{\lambda_{99}}\cdot\left(\frac{\lambda_{38}\cdot\lambda_{89}}{\lambda_{88}}+\lambda_{39}\right)\right]\right\}+\frac{\lambda_{36}\cdot\lambda_{67}}{\lambda_{66}\cdot\lambda_{77}}+\frac{\lambda_{34}}{\lambda_{44}}+\frac{\lambda_{35}}{\lambda_{55}}}{1+\left\langle\frac{\lambda_{12}}{\lambda_{11}\cdot\lambda_{22}}\left\{\left(\frac{\lambda_{41}\cdot\lambda_{34}}{\lambda_{44}}\right)+\left[\frac{\lambda_{91}}{\lambda_{99}}\cdot\left(\frac{\lambda_{38}\cdot\lambda_{89}}{\lambda_{88}}+\lambda_{39}\right)\right]\right\}\right\rangle\cdot\left(1+\frac{\lambda_{12}}{\lambda_{22}}\right)+\frac{\lambda_{34}}{\lambda_{44}}+\frac{\lambda_{35}}{\lambda_{55}}+\frac{\lambda_{36}}{\lambda_{66}}\cdot\left(1+\frac{\lambda_{67}}{\lambda_{77}}\right)+\frac{\lambda_{38}}{\lambda_{88}}\cdot\left(1+\frac{\lambda_{89}}{\lambda_{99}}\right)+\frac{\lambda_{39}}{\lambda_{99}}}. \tag{22}$$

For data obtained from examinations of the operational use process within a real transport system, there were determined intensity values λ_{ij} of the process state changes and values of boundary probabilities p_i^* of stay in Markov's process states $X(t)$, and next, using the above formulas, values of the transport system technical object availability (Tab. 1).

Tab. 1. Availability values of transport means used in a system of a city bus transport

$G_{OT(1)}^{24}$	$G_{OT(2)}^{24}$	$G_{OT(3)}^{24}$
0.8428	0.8553	0.8607

5. Conclusions

Availability of technical objects (transport means) used in a real city bus transport system, determined on the basis of Markov's model of the operational use process, is equal to the sum of boundary probabilities p_i^* of stays in the process states considered as the technical object availability states. Values of p_i^* boundary probabilities of the analyzed Markov's process depend directly on values of λ_{ij} intensities of the process state changes and indirectly on values of T_{ij} conditional times of stays in particular states of the process and on values of n_{ij} number of entries to particular states of the process. This proves that the change values of T_{ij} times and n_{ij} number of entries into the states, of the process results in the change of G_{OT} availability value of the transport system technical objects. Increase in the values of T_{ij} conditional times and n_{ij} number of entries to the process states, regarded as the technical object availability states, causes increase in the technical object availability.

In this work values of the technical object availability states for three distinguished levels have been determined:

- for the first level; availability $G^{24}_{OT(1)}$ is determined for objects which are serviceable and equipped,
- for the second level; availability $G^{24}_{OT(2)}$ is determined additionally for serviceable technical objects, which should be equipped with, eg. fuel, however, the process of equip will be realized in time shorter than the reserve time intended for equipping technical objects without losing a ride, that is, in the time between rides, according to the time-schedule,
- for the third level, availability $G^{24}_{OT(3)}$ is additionally determined for unserviceable technical objects, which were damaged on the hardstand of the bus depot during waiting for the transport task accomplishment or on the route while carrying out the transport task, however, they were serviceability assured by specialists from the bus servicing depot or breakdown truck units in time shorter than the reserve time intended for serviceability assurance without losing a ride, that is, between rides, according to the time-schedule.

Availability of technical objects (transport means) depends on many factors, and its value rise can be obtained by:

- in case of the first level:
  - use of technical objects with higher reliability,
  - use of the proper number of standby objects,
  - increase of effectiveness of serviceability assurance processes realized on the stands of the logistic system,
- in case of the second level:
  - adjustment (increase) of the number of stands of the refuelling subsystem,
  - use of refueling subsystem stands with higher efficiency,
- in case of the third level:
  - adjustment (increase) of the number of breakdown truck units,
  - equipment of the breakdown truck unit with tools and devices of higher efficiency and wider range of application,
  - increase of effectiveness of the serviceability assurance process realized by breakdown truck units,
  - use of technical objects with higher serviceability and adjustability.

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